Enrollment No: _______________________ **Exam Seat No:**_______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Discrete Mathematics

Subject Code : 4SC05DMC1 **Branch :**B.Sc.(Mathematics)

Semester : 5 **Date :** 09/12/2015 **Time :** 2:30 To 5:30 **Marks :** 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions (2 Marks each)** **(14)**

a) Define Lattice and give an example of it.

b) Draw the Hasse diagram of $\langle L^2, \leq \rangle$; where $L = \{0,1\}$

c) Define: (i) Complement of Fuzzy subset (ii) Characteristic function

d) Find the least and greatest element in the POSET $\langle Z^+, D \rangle$, if they exist.

e) Define: Atom and find all atoms of Boolean algebra $\langle S_{30}, D \rangle$.

f) Find the complements of every element of the lattice $\langle S_n, D \rangle$ for $n = 12$

g) Show that $a * a = a$ using absorption property.

Attempt any four questions from Q-2 to Q-8

Q-2 State and prove Stone's representation theorem. **(14)**

Q-3 **Attempt all questions**

A Obtain the 'Sum of Product' and 'Product of Sum' canonical forms of the Boolean expression in three variables $(x_1 * x_3) \oplus \left(x_1' * x_2\right) \oplus (x_2 * x_3)$. **(07)**

B Define direct product of two lattices and show that direct product of two lattices is also a lattice. **(07)**

Q-4 **Attempt all questions**

A For the POSET $\langle \{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,3,4\},\{2,3,4\}\}, \subseteq \rangle$, **(07)**

1) Draw the Hasse diagram.

2) Find maximal elements and minimal elements

3) Find Greatest element and least element, if exists

4) Find Lower bounds of $\{1,3,4\}$ and $\{2,3,4\}$

5) Find Upper bounds of $\{2,4\}$ and $\{3,4\}$

B Let (B, $*$, $\oplus$, ', 0, 1) be a Boolean algebra prove that **(07)**

$a = b \Leftrightarrow (a * b') \oplus (a' * b) = 0$

Q-5 **Attempt all questions**

A Let (L , $\leq$) be a lattice in which $*$ and $\oplus$ denote operations of meet and join. Then for any a, b $\in$ L prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$. **(07)**

B Obtain cube array representation for Boolean expression $h(x,y,z) = xy + y' + z'$ **(07)**

Q-6 **Attempt all questions**

A Obtain circuit diagram representation for the Boolean expression **(07)**

$$g(x_1,x_2,x_3) = [x_3(x_1+x_2)] + (x_1x_2') + (x_1'x_2) + (x_1'x_2'x_3')$$

Hence find minimal Sum of Product form of it.

B Obtain Karnaugh map representation for the Boolean map **(07)**

$f: B^3 \rightarrow B, f(x,y,z) = x + (yz')$

Q-7 **Attempt all questions**

A Determine whether the Boolean expressions given below are equivalent from their valuation tables. **(07)**

$$\alpha(x_1,x_2,x_3) = (x_1 \oplus x_3)' \oplus (x_1' * x_3); \quad \beta(x_1,x_2,x_3) = x_1' \oplus (x_1' * x_2' * x_3)$$

B Let $E = \{a,b,c,d,e\}$, $\underset{\sim}{A} = \{(a,0.3),(b,0.8),(c,0.5),(d,0.1),(e,0.9)\}$ **(07)**

$\underset{\sim}{B} = \{(a,0.7),(b,0.6),(c,0.4),(d,0.2),(e,0.1)\}$

Find: (1) $\underset{\sim}{A} \cup \underset{\sim}{B}$ (2) $\underset{\sim}{A} \cap \underset{\sim}{B}$ (3) $\underset{\sim}{A} \cdot \underset{\sim}{B}$ (4) $\underset{\sim}{A} \hat{+} \underset{\sim}{B}$ (5) $\underset{\sim}{A} - \underset{\sim}{B}$ (6) $\underset{\sim}{B}'$ (7) $(\underset{\sim}{A}')'$

Q-8 **Attempt all questions**

A State De Morgan's Laws for fuzzy subsets and prove any one. **(07)**

B Simplify the circuit given in following figure using Boolean identities. **(07)**